SCHINZEL ORDERINGS IN FUNCTION FIELDS

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In 1968, A. Schinzel considered the following problem: let K be a number field, O_K be its ring of integers. Does there exist a sequence $(a_n)_{n\in\mathbb{N}}$ of elements of O_K such that the set $\{a_1, \cdots, a_{\mathcal{N}(\mathcal{I})}\}$ forms a complete system of representants of $O_K/\mathcal{I}$ for any integral ideal $\mathcal{I}$ of O_K? Such a sequence is now called *Schinzel ordering of* O_K. We shall give some results about the existence of a Schinzel ordering in the setting of the function fields.

We shall also observe connections with the notion of *Newton ordering*. Let D be an integral domain D whose quotient field is K. Denote by $\mathrm{Int}(D)$ the D-module of integer-valued polynomials over D, that is

$$\mathrm{Int}(D) = \{f \in K[X] \mid f(D) \subseteq D\}.$$

A sequence $(b_n)_{n\in\mathbb{N}}$ of elements of D is a *Newton ordering of* D if the sequence of polynomials

$$P_n(X) = \prod_{k=0}^{n-1} \frac{X - b_k}{b_n - b_k}$$

is a basis of $\mathrm{Int}(D)$.

In the case where D is the ring of integers of a global field K, the notion of Newton ordering coincides with *simultaneous ordering* introduced by M. Bhargava. We answer to a question raised by D. Thakur (corresponding to an analog of Schinzel problem) on the existence of a simultaneous ordering for a certain class of function fields.